CHAPTER - 9
RATIONAL NUMBERS

EXERCISE 9.1

Q1. List five rational numbers between :

(i) -1 & 0 (ii) -2 & -1

(iii) $\frac{-4}{5}$ & $\frac{-2}{3}$ (iv) $\frac{-1}{2}$ and $\frac{2}{3}$

→ (i) -1 and 0

Converting each of rational numbers as a denominator $5+1=6$ we have

$$-1 = \frac{-1\times 6}{6} = \frac{-6}{6} \quad \text{and} \quad 0 = \frac{0\times 6}{6} = \frac{0}{6}$$

$$\therefore \frac{-6}{6} < \frac{-5}{6} < \frac{-4}{6} < \frac{-3}{6} < \frac{-2}{6} < \frac{-1}{6} < \frac{0}{6}$$

$$\therefore -1 < \frac{-5}{6}, \frac{-4}{6}, \frac{-3}{6}, \frac{-2}{6}, \frac{-1}{6} < 0$$

(ii) -2 and -1

Converting each of rational numbers as a denominator 6, we have

$$-2 = \frac{-2\times 6}{6} = \frac{-12}{6} \qquad -1 = \frac{-1\times 6}{6} = \frac{-6}{6}$$

$$\therefore \frac{-12}{6} < \frac{-11}{6}, \frac{-10}{6}, \frac{-9}{6}, \frac{-8}{6}, \frac{-7}{6} < \frac{-6}{6}$$

Required rational numbers are $\frac{-11}{6}, \frac{-10}{6}, \frac{-9}{6}, \frac{-8}{6}, \frac{-7}{6}$

(iii) $\frac{-4}{5}$ and $\frac{-2}{3}$

Converting each rational numbers as 15, we have

$\frac{-4}{5} = \frac{-4\times3}{5\times3} = \frac{-12}{15}$, $\frac{-2}{3} = \frac{-2\times5}{3\times5} = \frac{-10}{15}$

Since there is only one rational number, will find the equivalent fractions.

$\frac{-12}{15} = \frac{-12\times3}{15\times3} = \frac{-36}{45}$, $\frac{-10}{15} = \frac{-10\times3}{15\times3} = \frac{-30}{45}$

$\therefore \frac{-36}{45} < \frac{-35}{45}, \frac{-34}{45}, \frac{-33}{45}, \frac{-32}{45}, \frac{-31}{45} < \frac{-30}{45}$

$\therefore$ Required in between rational numbers are

$\frac{-35}{45}, \frac{-34}{45}, \frac{-33}{45}, \frac{-32}{45}, \frac{-31}{45}$

(iv) $\frac{-1}{2}$ and $\frac{2}{3}$

Converting each of rational numbers to their equivalent rational numbers we have,

$\frac{-1}{2} = \frac{-1\times18}{2\times18} = \frac{-18}{36}$; $\frac{2}{3} = \frac{2\times12}{3\times12} = \frac{24}{36}$

$\therefore \frac{-18}{36}$ & $\frac{-18}{36} < \frac{-17}{36} < \frac{-16}{36} < \frac{-15}{36} < \frac{-14}{36} < \frac{-13}{36} \ldots < \frac{24}{36}$

Required in between rational numbers are

$\frac{-17}{36}, \frac{-16}{36}, \frac{-15}{36}, \frac{-14}{36}, \frac{-13}{36}$

Q2 Write four more rational numbers in each of the following patterns:

(i) $\frac{-3}{5}, \frac{-6}{10}, \frac{-9}{15}, \frac{-12}{20}, \ldots\ldots$

→ Given pattern is

$\frac{-3}{5} = \frac{-3\times1}{5\times1} = \frac{-3}{5}$

$\frac{-6}{10} = \frac{-3\times2}{5\times2} = \frac{-6}{10}$

$\frac{-9}{15} = \frac{-3\times3}{5\times3} = \frac{-9}{15}$

$\frac{-12}{20} = \frac{-3\times4}{5\times4} = \frac{-12}{20}$

Preceeding with the pattern

$\frac{-3\times5}{5\times5} = \frac{-15}{25}$

$\frac{-3\times6}{5\times6} = \frac{-18}{30}$

$\frac{-3\times7}{5\times7} = \frac{-21}{35}$

$\frac{-3\times8}{5\times8} = \frac{-24}{40}$

Hence, required rational numbers are

$\frac{-15}{25}, \frac{-18}{30}, \frac{-21}{35}, \frac{-24}{40}$

(ii) $\frac{-1}{4}, \frac{-2}{8}, \frac{-3}{12}, \ldots$

→ Given patterns is

$\frac{-1}{4} = \frac{-1\times1}{4\times1} = \frac{-1}{4}$

$\frac{-2}{8} = \frac{-1\times2}{4\times2} = \frac{-2}{8}$

$\frac{-3}{12} = \frac{-1\times3}{4\times3} = \frac{-3}{12}$

Preceeding with the pattern

$\frac{-1\times4}{4\times4} = \frac{-4}{16}$

$\frac{-1\times5}{4\times5} = \frac{-5}{20}$

$\frac{-1\times6}{4\times6} = \frac{-6}{24}$

$\frac{-1\times7}{4\times7} = \frac{-7}{28}$

Hence, required rational numbers are

$\frac{-4}{16}, \frac{-5}{20}, \frac{-6}{24}, \frac{-7}{28}.$

(iii) $\frac{-1}{6}, \frac{2}{-12}, \frac{3}{-18}, \frac{4}{-24} \ldots$

→ Given pattern is

$\frac{-1}{6} = \frac{-1\times1}{6\times1} = \frac{-1}{6}$

$\frac{-2}{12} = \frac{-1\times2}{6\times2} = \frac{-2}{12}$

$\frac{-3}{18} = \frac{-1\times3}{6\times3} = \frac{-3}{18}$

$\frac{-4}{24} = \frac{-1\times4}{6\times4} = \frac{-4}{24}$

Preceeding with the same pattern

$\frac{-1\times5}{6\times5} = \frac{-5}{30}$

$\frac{-1\times6}{6\times6} = \frac{-6}{36}$

$\frac{-1\times7}{6\times7} = \frac{-7}{42}$

$\frac{-1\times8}{6\times8} = \frac{-8}{48}$

Hence, required rational numbers are

$\frac{-5}{30}, \frac{-6}{36}, \frac{-7}{42}, \frac{-8}{48}$

(ix) $\frac{-2}{3}, \frac{2}{-3}, \frac{4}{-6}, \frac{6}{-9}$ ....

→ Given pattern is

$\frac{-2}{3} = \frac{-2\times1}{3\times1} = \frac{-2}{3}$

$\frac{2}{-3} = \frac{2\times1}{-3\times1} = \frac{2}{-3}$

$\frac{4}{-6} = \frac{-4}{6} = \frac{-2\times2}{3\times2}$

$\frac{-6}{-9} = \frac{-2\times3}{3\times3} = \frac{-6}{9}$

Preceeding with the pattern

$\frac{-2\times4}{3\times4} = \frac{-8}{12}$

$\frac{-2\times5}{3\times5} = \frac{-10}{15}$

$\frac{-2\times6}{3\times6} = \frac{-12}{18}$

$\frac{-2\times7}{3\times7} = \frac{-14}{21}$

Hence, required rational numbers are

$\frac{-8}{12}, \frac{-10}{15}, \frac{-12}{18}, \frac{-14}{21}$.

Q3. Give four rational numbers equivalent to:

(i) $\frac{-2}{7}$ (ii) $\frac{5}{-3}$ (iii) $\frac{4}{9}$

→ (i) $\frac{-2}{7} = \frac{-2\times2}{7\times2} = \frac{-4}{14}$ (ii) $\frac{5}{-3} = \frac{5\times2}{-3\times2} = \frac{10}{-6}$

$\frac{-2}{7} = \frac{-2\times3}{7\times3} = \frac{-6}{21}$ $\frac{5}{-3} = \frac{5\times3}{-3\times3} = \frac{15}{-9}$

$\frac{-2}{7} = \frac{-2\times4}{7\times4} = \frac{-8}{28}$ $\frac{5}{-3} = \frac{5\times4}{-3\times4} = \frac{20}{-12}$

$\frac{-2}{7} = \frac{-2\times5}{7\times5} = \frac{-10}{35}$ $\frac{5}{-3} = \frac{5\times5}{-3\times5} = \frac{25}{-15}$

(iii) $\frac{4}{9} = \frac{4\times2}{9\times2} = \frac{8}{18}$

$\frac{4}{9} = \frac{4\times3}{9\times3} = \frac{12}{27}$

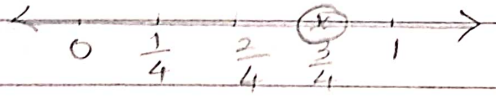
$\frac{4}{9} = \frac{4\times4}{9\times4} = \frac{16}{36}$

$\frac{4}{9} = \frac{4\times5}{9\times5} = \frac{20}{45}$

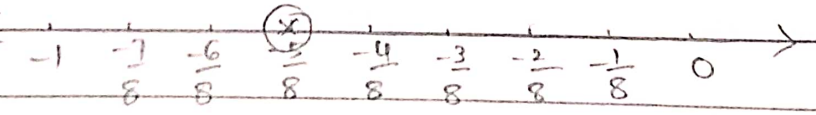
Q4. Draw the number line & represent the following ration numbers on it:

(i) $\frac{3}{4}$ (ii) $\frac{-5}{8}$ (iii) $\frac{-7}{4}$ (iv) $\frac{7}{8}$

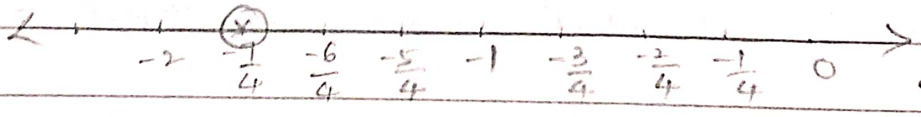
→ (i) 3/4



(ii) $\frac{-5}{8}$



(iii) $\frac{-7}{4}$



(iv) $\frac{7}{8}$

Q5. The points P, Q, R, S, T, U, A, B on the number line are such that, TR = RS = SU & AP = PQ = QB. Name the rational numbers represented by P, Q, R, S.

→ Rational numbers represented by P, Q, R, S are $\frac{7}{3}, \frac{8}{3}, \frac{-4}{3}, \frac{-5}{3}$ resp.

Q6. Which of the following pairs represent the same rational number?

→ (i) $\frac{-7}{21}$ and $\frac{3}{9}$

converting each of the rational numbers to its lowest form

$$\frac{-7}{21} = \frac{-7 \div 7}{21 \div 7} = \frac{-1}{3} \qquad \frac{3}{9} = \frac{3 \div 3}{9 \div 3} = \frac{1}{3}$$

Since $\frac{-1}{3} \neq \frac{1}{3}$; $\frac{-7}{21}$ and $\frac{3}{9}$ do not represent the same rational numbers.

(ii) $\frac{-16}{20}$ and $\frac{20}{-25}$

→ converting each of the rational numbers to its lowest form

$$\frac{-16}{20} = \frac{-16 \div 4}{20 \div 4} = \frac{-4}{5} \qquad \frac{20}{-25} = \frac{20 \div 5}{-25 \div 5} = \frac{4}{-5} = \frac{-4}{5}$$

Since $\frac{-4}{5} = \frac{-4}{5}$; $\frac{-16}{20}$ & $\frac{20}{-25}$ represent the same rational numbers.

(iii) $\frac{-2}{-3}$ and $\frac{2}{3}$

→ Since the given rational numbers are in its lowest form, & numerator & denominator are same

$\therefore \frac{-2}{-3}$ & $\frac{2}{3}$ represent the same rational numbers.

(iv) $\frac{-3}{5}$ and $\frac{-12}{20}$

→ Converting $-12/20$ in its lowest form

$$\frac{-12}{20} = \frac{-12 \div 4}{20 \div 4} = \frac{-3}{5}$$

Since $\frac{-3}{5} = \frac{-3}{5}$; $\frac{-3}{5}$ & $\frac{-20}{25}$ represent the same rational numbers.

(v) $\frac{8}{-5}$ and $\frac{-24}{15}$

→ Converting $-24/15$ in its lowest form

$$\frac{-24}{15} = \frac{-24 \div 3}{15 \div 3} = \frac{-8}{5} = \frac{8}{-5}$$

Since, $\frac{8}{-5} = \frac{8}{-5}$; $\frac{8}{-5}$ and $\frac{-24}{15}$ represent the same rational numbers.

(vi) $\frac{1}{3}$ and $\frac{-1}{9}$

→ Here both are in their lowest form

$\therefore \frac{1}{3} \neq \frac{-1}{9}$, they do not represed the same rational numbers.

Q7. Rewrite the following rational numbers in the simplest form:

(i) $\frac{-8}{6} = \frac{-8 \div 2}{6 \div 2} = \frac{-4}{3}$

(ii) $\frac{25}{45} = \frac{25 \div 5}{45 \div 5} = \frac{5}{9}$

(iii) $\frac{-44}{72} = \frac{-44 \div 4}{72 \div 4} = \frac{-11}{18}$

(iv) $\frac{-8}{10} = \frac{-8 \div 2}{10 \div 2} = \frac{-4}{5}$

Q8. Fill in the boxes with the correct symbol out of $>$, $<$ and $=$:

(i) $\frac{-5}{7} \square \frac{2}{3}$

$\rightarrow \frac{-5 \times 3}{7 \times 3} = \frac{-15}{21}$;

$\frac{2 \times 7}{3 \times 7} = \frac{14}{21}$

$\therefore \frac{-15}{21} \boxed{<} \frac{14}{21}$

$\therefore \frac{-5}{7} \boxed{<} \frac{2}{3}$

(ii) $\frac{-4}{5} \square \frac{-5}{7}$

$\frac{-4 \times 7}{5 \times 7} = \frac{-28}{35}$

$\frac{-5 \times 5}{7 \times 5} = \frac{-25}{35}$

$\frac{-28}{35} \boxed{<} \frac{-25}{35}$

$\therefore \frac{-4}{5} \boxed{<} \frac{-5}{7}$

(iii) $\frac{-7}{8} \square \frac{14}{-16}$

$\frac{-7 \times 2}{8 \times 2} = \frac{-14}{16}$

$\frac{14 \times (-1)}{-16 \times (-1)} = \frac{-14}{16}$

$\frac{-14}{16} \boxed{=} \frac{-14}{16}$

$\therefore \frac{-7}{8} \boxed{=} \frac{14}{-16}$

(iv) $\frac{-8}{5} \square \frac{-7}{4}$

$\frac{-8 \times 4}{5 \times 4} = \frac{-32}{20}$

$\frac{-7 \times 5}{4 \times 5} = \frac{-35}{20}$

$\frac{-32}{20} \boxed{>} \frac{-35}{20}$

$\therefore \frac{-8}{5} \boxed{>} \frac{-7}{4}$

(v) $\frac{1}{-3} \square \frac{-1}{4}$

$\frac{1\times4}{-3\times4} = \frac{4}{-12}$

$\frac{-1\times3}{4\times3} = \frac{-3}{12} = \frac{3}{-12}$

$\frac{4}{-12} > \frac{3}{-12}$

$\therefore \frac{1}{-3} > \frac{-1}{4}$

(vi) $\frac{5}{-11} \square \frac{-5}{11}$

$\frac{5\times11}{-11\times11} = \frac{55}{-121}$

$\frac{-5\times11}{11\times11} = \frac{-55}{121}$

$\frac{55}{121} = \frac{-55}{121}$

$\therefore \frac{5}{-11} = \frac{-5}{11}$

(vii) $0 > \frac{-1}{6}$ Because 0 is greater than every negative number.

Q9. Which is greater in each of the following:

(i) $\frac{2}{3}, \frac{5}{2}$ [LCM of 3, 2 = 6]

$\rightarrow \frac{2\times2}{3\times2} = \frac{4}{6}$; $\frac{5\times3}{2\times3} = \frac{15}{6}$

$\frac{4}{6} < \frac{15}{6} \Rightarrow \frac{2}{3} < \frac{5}{2}$

(ii) $\frac{-5}{6}, \frac{-4}{3}$

$\rightarrow \frac{-5}{6} = \frac{-5\times1}{6\times1} = \frac{-5}{6}$ [LCM of 6, 3 = 6]

$\frac{-4}{3} = \frac{-4\times2}{3\times2} = \frac{-8}{6}$

$\frac{-5}{6} > \frac{-8}{6} \Rightarrow \frac{-5}{6} > \frac{-4}{3}$

(iii) $\frac{-2}{4}, \frac{2}{-3}$

→ $\frac{-2 \times -3}{4 \times -3} = \frac{6}{-12}$; $\frac{2 \times 4}{-3 \times 4} = \frac{8}{-12}$ [LCM of 4, -3 = 12]

$\frac{6}{-12} > \frac{8}{-12} \Rightarrow \frac{2}{-3} > \frac{-3}{4}$

(iv) $\frac{-1}{4}, \frac{1}{4}$

→ $\frac{-1}{4} < \frac{1}{4}$

(v) $\frac{-22}{7}, \frac{-24}{5}$

→ $\frac{-23}{7}, \frac{-19}{5}$

$\frac{-22 \times 5}{7 \times 5} = \frac{-115}{35}$; $\frac{-19 \times 7}{5 \times 7} = \frac{-133}{35}$

$\frac{-115}{35} > \frac{-133}{35} \Rightarrow \frac{-22}{7} > \frac{-24}{5}$

Q10. Write the following rational numbers in ascending order :

(i) $\frac{-3}{5}, \frac{-2}{5}, \frac{-1}{5}$

→ Here denominators are same

$-3 < -2 < -1$

$\therefore \frac{-3}{5} < \frac{-2}{5} < \frac{-1}{5}$

(ii) $\frac{-1}{3}, \frac{-2}{9}, \frac{-4}{3}$

→ LCM of 3, 9, 3 = 9

$\frac{-1\times3}{3\times3} = \frac{-3}{9}$; $\frac{-2\times1}{9\times1} = \frac{-2}{9}$; $\frac{-4\times3}{3\times3} = \frac{-12}{9}$

$\frac{-12}{9} < \frac{-3}{9} < \frac{-2}{9} \Rightarrow \frac{-4}{3} < \frac{-1}{3} < \frac{-2}{9}$

(iii) $\frac{-3}{7}, \frac{-3}{2}, \frac{-3}{4}$

→ LCM of 7, 2, 4 = 28

$\frac{-3\times4}{7\times4} = \frac{-12}{28}$; $\frac{-3\times14}{2\times14} = \frac{-42}{28}$; $\frac{-3\times7}{4\times7} = \frac{-21}{28}$

$\frac{-42}{28} < \frac{-21}{28} < \frac{-12}{28} \Rightarrow \frac{-3}{2} < \frac{-3}{4} < \frac{-3}{7}$

EXERCISE 9.2

Q1. Find the sum:

(i) $\frac{5}{4} + \left(\frac{-1}{4}\right) = \frac{5+(-1)}{4} = \frac{-6}{4} = \frac{-3}{2}$

(ii) $\frac{5}{3} + \frac{3}{5}$

LCM of 3, 5 = 15

$\frac{5}{3} + \frac{3}{5} = \frac{(5\times5) + (3\times3)}{15} = \frac{25+9}{15} = \frac{34}{15}$

(iii) $\frac{-9}{10} + \frac{22}{15}$

LCM of 10, 15 = 30

$\frac{-9}{10} + \frac{22}{15} = \frac{(-9\times3) + (22\times2)}{30} = \frac{-27+44}{30} = \frac{17}{30}$

(iv) $\frac{-3}{-811} + \frac{5}{9} = \frac{3}{11} + \frac{5}{9}$

LCM of 11, 9 = 99

$\frac{3}{11} + \frac{5}{9} = \frac{(3\times9) + (5\times11)}{99} = \frac{27+55}{99} = \frac{82}{99}$

(v) $\frac{-8}{19} + \frac{(-2)}{57} = \frac{-8}{19} - \frac{2}{57}$

LCM of 19, 57 = 57

$\frac{-8\times3}{19\times3} = \frac{-24}{57}$ $\quad$ $\frac{-2}{57} = \frac{-2\times1}{57\times1} = \frac{-2}{57}$

$\frac{-24 + (-2)}{57} = \frac{-26}{57}$

(vi) $\frac{-2}{3} + 0 = \frac{-2}{3}$

(vii) $-2\frac{1}{3} + 4\frac{3}{5}$

$-\frac{7}{3} + \frac{23}{5}$

LCM of 3, 5 = 15

$-\frac{7}{3} + \frac{23}{5} = \frac{(-7\times5) + (23\times3)}{15} = \frac{-35+69}{15} = \frac{34}{15}$

Q2. Find

(i) $\frac{7}{24} - \frac{17}{36}$

LCM of 24 and 36 is 72

$\frac{7}{24} - \frac{17}{36} = \frac{(7\times3) - (17\times2)}{72} = \frac{21-34}{72} = \frac{-13}{72}$

(ii) $\frac{5}{63} - \left(\frac{-6}{21}\right) = \frac{5}{63} + \frac{6}{21}$

LCM of 63, 21 is 63.

$\frac{5}{63} + \frac{6}{21} = \frac{(5\times1) + (6\times3)}{63} = \frac{5+18}{62} = \frac{23}{63}$

(iii) $\frac{-6}{13} - \left(\frac{-7}{15}\right) = \frac{-6}{13} + \frac{7}{15}$

LCM of 13, 15 is 195

$\frac{-6}{13} + \frac{7}{15} = \frac{(-6\times15) + (7\times13)}{195} = \frac{(-90) + (91)}{195} = \frac{1}{195}$

(iv) $\frac{-3}{8} - \frac{7}{11}$

LCM of 8, 11 is 88.

$\frac{-3}{8}, \frac{-7}{11} = \frac{(-3\times11) - (7\times8)}{88} = \frac{-33-56}{88} = \frac{-89}{88}$

(v) $-2\frac{1}{9} - 6 = \frac{-19}{9} - \frac{6}{1}$

LCM of 9, 1 = 9

$\frac{-19}{9} - \frac{6}{1} = \frac{(-19\times1) - (6\times9)}{9} = \frac{-19-54}{9} = \frac{-73}{9}$

Q3. Find the product:

(i) $\frac{9}{2} \times \left(\frac{-7}{4}\right) = \frac{9\times(-7)}{2\times4} = \frac{-63}{8}$

(ii) $\frac{3}{10} \times (-9) = \frac{3\times(-9)}{10} = \frac{-27}{10}$

(iii) $\frac{-6}{5} \times \frac{9}{11} = \frac{-6 \times 9}{5 \times 11} = \frac{-54}{55}$

(iv) $\frac{3}{7} \times \left(\frac{-2}{5}\right) = \frac{3 \times (-2)}{7 \times 5} = \frac{-6}{35}$

(v) $\frac{3}{11} \times \frac{2}{5} = \frac{3 \times 2}{11 \times 5} = \frac{6}{55}$

(vi) $\frac{3}{-5} \times \frac{-5}{3} = \frac{3 \times (-5)}{(-5) \times 3} = \frac{-15}{-15} = 1$

Q4. Find the value of :

(i) $(-4) \div \frac{2}{3} = -\frac{4}{1} \times \frac{3}{2} = -6$

(ii) $\frac{-3}{5} \div 2 = \frac{-3}{5} \times \frac{1}{2} = \frac{-3}{10}$

(iii) $\frac{-4}{5} \div (-3) = \frac{-4}{5} \times \frac{1}{-3} = \frac{-4}{-15}$

(iv) $\frac{-1}{8} \div \frac{3}{4} = \frac{-1}{2} \times \frac{1}{3} = \frac{-1}{6}$

(v) $\frac{-2}{13} \div \frac{1}{7} = \frac{-2}{13} \times \frac{7}{1} = \frac{-14}{13}$

(vi) $\frac{-7}{12} \div \left(\frac{-2}{13}\right) = \frac{-7}{12} \times \frac{13}{-2} = \frac{-91}{-24} = \frac{91}{24}$

(vii) $\frac{3}{13} \div \left(\frac{-4}{5}\right) = \frac{3}{1} \times \frac{5}{-4} = \frac{-15}{4}$